ENERGY LOSSES IN A TWO-PHASE FLOW DUE TO MECHANICAL INTERACTION OF THE PHASES

M. P. Anisimova and E. V. Stekol'shchikov

Inzhenerno-Fizicheskii Zhurnal, Vol. 15, No. 3, pp. 436–443, 1968

UDC 532.529.5

Relations are obtained for calculating the energy loss coefficient due to friction at the phase interface in a one-dimensional two-phase flow. Calculated values of the loss coefficient as a function of a number of dimensionless criteria are presented for the case of linear variation of the gas velocity as a function of the coordinate.

In a two-phase as compared with a one-phase flow there are a considerable number of additional sources of kinetic energy losses. These include: a) losses due to interphase heat transfer at a finite phase temperature difference, b) losses due to phase transition at a finite value of the concentration difference, c) losses due to friction at the phase interfaces, d) losses in condensation shocks, e) kinetic energy losses associated with the work done by surface tension forces in connection with the curvature of the phase interface, f) energy losses associated with the circulation of liquid inside a particle and with the repeated changes in the shape of a liquid droplet flattened by the flow during rotation, etc. Below, we analyze the energy losses in a one-dimensional two-phase flow associated with friction at the phase interface. Two coordinate systems are introduced: a system fixed in space (absolute) and the moving system of the center of gravity of the particle.

The elementary work done by the gas phase in absolute motion

$$dA_1 = N_1 d\tau_s = (N_2 + N_3)\, d\tau_s. \tag{1}$$

The useful work done by the gas in acceleration or deceleration is equal to the change in the kinetic energy of the liquid (solid) particle:

$$N_2 d\tau_s = Fdz = dE_s = (1 - x)\, c_s dc_s. \tag{2}$$

The equations of motion of particles in gaseous media are usually obtained from (2). For a particle of arbitrary shape

$$\frac{d\nu}{d\varphi} = \frac{(\nu - \nu_{v1})(\nu_{v2} - \nu)}{\nu}. \tag{3}$$

The work done by the gas in relative motion is equal to the sum of the amounts of work done in the process of reversible and irreversible energy transfer:

$$F_2(c_v - c_s)\, d\tau_s, \quad (4) \qquad F_1(c_v - c_s)\, d\tau_s. \quad (5)$$

The total work done by the gas in accelerating or decelerating the particles

$$dA = dE_s + dA_f = dA_1 - F_2(c_v - c_s)\, d\tau_s, \tag{6}$$

where

$$dA_f = |F_1(c_v - c_s)\, d\tau_s|. \tag{7}$$

If in (1) the elementary work done by the gas is calculated not for the time of particle motion ($d\tau_s$), but for an arbitrary time $d\tau$ not associated with the time taken by the liquid droplets or gas particles to traverse a given distance dz, then the relation for calculating the energy losses becomes more general than (7). It can be used for calculating A_f at any, including zero ($c_v = 0$ or $c_s = 0$) values of c_v and c_s.

We have

$$F_1 = (1-x)\left[\frac{(c_v)_0^2}{\Delta z}\right]\left[\frac{\bar{c}_v - \bar{c}_s}{R_1 \tau_g}\right], \tag{8}$$

$$F_2 = (1-x)\left[\frac{(c_v)_0^2}{\Delta z}\right]\left[\frac{R_2}{R_1}\ \bar{c}_v \frac{d\bar{c}_v}{d\bar{z}}\right]. \tag{9}$$

Transforming (7) with allowance for (8), we obtain

$$d\bar{A}_f = \left|(1-x)\ \frac{(1-\nu)^2}{R_1 \bar{\tau}_g \nu}\ d(\bar{c}_v)^2\right|. \tag{10}$$

At $(1 - x)/R_1\bar{\tau}_g$ = const

$$\bar{A}_f = \left|\frac{1-x}{R_1\bar{\tau}_g}\int\limits_1^{\bar{c}_v}\frac{(1-\nu)^2}{\nu}\,\bar{d}(\bar{c}_v)^2\right|. \tag{11}$$

We define the available kinetic energy of the two-phase flow at $\bar{z} = 0$ and any intermediate point ($0 < \bar{z} \leq 1$) as

$$E_0 = x\ \frac{(c_v)_0^2}{2} + (1-x)\ \frac{(c_s)_0^2}{2}, \tag{12}$$

$$E = x\ \frac{(c_v)^2}{2} + (1-x)\ \frac{(c_s)^2}{2} + A_f. \tag{13}$$

We estimate the economy of the process of kinetic energy transfer between the gas and the liquid in terms of the loss coefficient ζ

$$\zeta = \frac{A_f}{|E - E_0|} = \frac{\bar{A}_f}{\bar{A}_f + |x|(\bar{c}_v)^2 - 1| + (1-x)\,[\bar{c}_v\,\nu)^2 - \nu_0^2]\,|}. \tag{14}$$

It is important to determine $A_f/(1 - x)$ and ζ for certain cases of self-similar motion of particles in a gas flow.

a) If the vapor and liquid phases move independently of each other without interchanging momentum (nonequilibrium limit),

$$\nu = \nu_0/\bar{c}_v. \tag{15}$$

Substituting (15) into (11) and (14), we obtain

$$\bar{A}_f/(1-x) = 0 \tag{16}$$

and

$$\zeta = 0. \tag{17}$$

b) If $\nu_0 = \nu_{vi}$ = const (motion of a two-phase medium with zero degree of nonequilibrium), then

$$\frac{\bar{A}_f}{1-x} = \left|\frac{(1-\nu_{vi})^2}{R_1\bar{\tau}_g\nu_{vi}}\ (\bar{c}_v - 1)(\bar{c}_v + 1)\right| \tag{18}$$

and

$$\zeta = \frac{(1-x)(1-\nu_{vi})^2}{(1-x)(1-\nu_{vi})^2 + |R_1\bar{\tau}_g\nu_{vi}\,|x + (1-x)\,\nu_{vi}^2||}. \tag{19}$$

In case b) irrespective of the magnitude and sign of $d\bar{c}_v/d\bar{z}$ relaxation processes associated with interphase momentum transfer cannot take place in the two-phase flow. This is the only particular case of motion when the slip factor ν and the loss coefficient ζ do not depend on the coordinate $\bar{z}$.

If $\nu_0 \neq \nu_{vi}$, Eqs. (18) and (19) give the limiting values of $\bar{A}_f/(1 - x)$ and ζ as $z \to \infty$. It is clear from (19) that the Stokes number τ_g and $d\bar{c}_v/d\bar{z}$ have, qualitatively and quantitatively, the same effect on the limiting coefficient ζ. The maximum value of ζ in the case of convergent-channel flow lies in the range $\bar{\tau}_g = 1$–10.

If the dimensionless vapor velocity depends linearly on the $\bar{z}$ coordinate

$$\bar{c}_v = (\varphi_v - 1)\bar{z} + 1, \tag{20}$$

we find that, in the case of a convergent-channel and zero-gradient vapor flow, $\bar{c}_V$ and ν are related by the following expression:

$$(\bar{c}_v)^2 = \left[\frac{\nu_0 - \nu_{v1}}{\nu - \nu_{v1}}\right]^{\left(\frac{\sqrt{\Delta}-1}{\sqrt{\Delta}}\right)} \left[\frac{\nu_0 - \nu_{v2}}{\nu - \nu_{v2}}\right]^{\left(\frac{1+\sqrt{\Delta}}{\sqrt{\Delta}}\right)}. \tag{21}$$

Substituting (21) into (11) and carrying out the necessary transformations, we obtain

$$\bar{A}_f/(1 - x) = \int_0^f (\bar{c}_v)^2 \, df, \tag{22}$$

where

$$f = 2\left[\frac{\nu_0 - \nu}{R_1 \bar{\tau}_g} + \frac{(1 - \nu_{v1})^2}{\sqrt{\Delta}} \ln \frac{\nu_0 - \nu_{v1}}{\nu - \nu_{v1}} - \right.$$
$$\left. - \frac{(1 - \nu_{v2})^2}{\sqrt{\Delta}} \ln \frac{\nu_0 - \nu_{v2}}{\nu - \nu_{v2}}\right].$$

If $\bar{c}_V - 1 \leq 10^{-3}$, then $\bar{A}_f \simeq (1 - x)f(\bar{c}_V)^2$.

The limiting value of ζ as $z \to 0$ can be calculated from the following equation obtained from (11), (14), and (21):

$$\zeta_0 = \lim_{z \to 0} \zeta = \frac{(1 - x)(1 - \nu_0)^2}{(1 - x)(1 - \nu_0)^2 + \nu_0 \,|\, x R_1 \bar{\tau}_g + (1 - x)(1 - \nu_0 + R_2 \bar{\tau}_g)|}. \tag{23}$$

The other limiting value: $\zeta_\infty = \lim_{z \to \infty} \zeta$ is determined from Eq. (19) above, if c_V is a monotonically varying function of the z-coordinate.

If $\nu_0 \leq 1$, then, other things being equal, as z varies from zero to infinity the coefficient ζ varies monotonically from ζ_0 to ζ_∞. In this case, ζ either decreases ($\nu_0 < \nu_{V1}$; $\zeta_0 > \zeta_\infty$) or increases ($\nu_0 > \nu_{V1}$; $\zeta_0 < \zeta_\infty$) or remains unchanged ($\nu_0 = \nu_{V1}$; $\zeta_0 = \zeta_\infty$). If $\nu_0 > 1$, then ζ also varies from ζ_0 to ζ_∞, but passes through a maximum. The presence of a maximum can easily be explained if one recalls that at the beginning of the particle's path $[(\bar{c}_V\nu)^2 - \nu_0^2] <$ < 0. At some value of z the terms $x[(\bar{c}_V)^2 - 1]$ and $(1 - x)\,[(\bar{c}_V\nu)^2 - \nu_0^2]$ in Eq. (14) become equal in magnitude, but remain opposite in sign. In this case, the coefficient ζ has a maximum value equal to 1. The value of $\bar{c}_V$ corresponding to $\zeta = \zeta_{max} = 1$ can be calculated by solving Eqs. (21) and

$$(\bar{c}_v)^2 = \frac{x + (1 - x)\nu_0^2}{x + (1 - x)\nu^2}. \tag{24}$$

Equation (24) was obtained from the relation $x[(\bar{c}_V)^2 - 1] + (1 - x)\,[(\bar{c}_V\nu)^2 - \nu_0^2] = 0$. By way of illustration, Fig. 1 presents the curves of ζ as a function of $(\bar{c}_V - 1)$ and x for $\nu_0 = 0.6$ and $\nu_0 = 2$. The calculations were based on Eq. (14). At $\nu_0 = 0.6$ and $\bar{\tau}_g = 0.1$ $\nu_0 < \nu_{V1}$. Consequently, as z increases, the coefficient ζ decreases (Fig. 1, I).

The dependence of ζ on $(\varphi_V - 1)$ is presented in Fig. 2 for three values of the Stokes number τ_g. It is clear from the graphs that an increase in $\varphi_V - 1$ leads to a decrease in ζ. For the conditions of Fig. 2 the greatest value of ζ corresponds to a zero-gradient flow.

For a zero-gradient vapor flow ($\varphi_V = 1$), as follows from (21),

$$\bar{z} = R_1 \tau_g \left[\nu_0 - \nu + \ln \frac{\nu_0 - 1}{\nu - 1}\right]. \tag{25}$$

Jointly transforming (11), (14) and (25), we obtain

$$\bar{A}_f/(1-x) = |2(\nu-\nu_0) - (\nu^2 - \nu_0^2)| \quad (26)$$

and

$$\zeta = \frac{|(1-\nu)+(1-\nu_0)|}{\nu+\nu_0+|(1-\nu)+(1-\nu_0)|}. \quad (27)$$

Equation (27) gives the limiting value to which ζ in Fig. 2 tends when $(\varphi_V - 1) \to 0$ and τ_g = const. The forms of dependence of ζ on $(\varphi_V - 1)$ are far from being exhausted by Fig. 2. In particular, if we set τ_g = const in (19) and (23), we obtain two other forms of the dependence of ζ on $(\varphi_V - 1)$ which, as distinct from the relations of Fig. 2, are extremal in character.

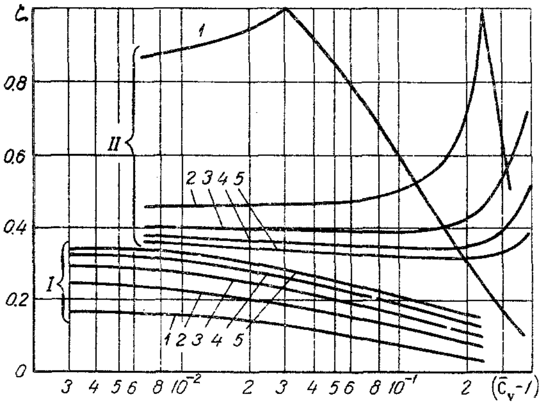

Fig. 1. Coefficient ζ as a function of $(\bar{c}_V - 1)$ and x for $\nu_0 = 0.6$ (I); $\nu_0 = 2$ (II); $\bar{\tau}_g = 0.1$; $\nu_{V1} = 0.9161$; $R_1 = 1$; $R_2 = 0$: 1) x = 0.9; 2) 0.8; 3) 0.7; 4) 0.6; 5) 0.5.

Certain forms of the dependence of ζ on ν_0 are presented in Fig. 3. Curve 6 is typical of small $\bar{c}_V - 1$ and $\bar{\tau}_g$. For $\bar{c}_V - 1 = 10^{-4}$; $\bar{\tau}_g = 10^{-4}$; x = 0.5–0.9 (Fig. 3) the coefficient ζ varies from 0 to 0.57. In this case the range of variation of $\zeta(\nu_0)$ is not maximal.

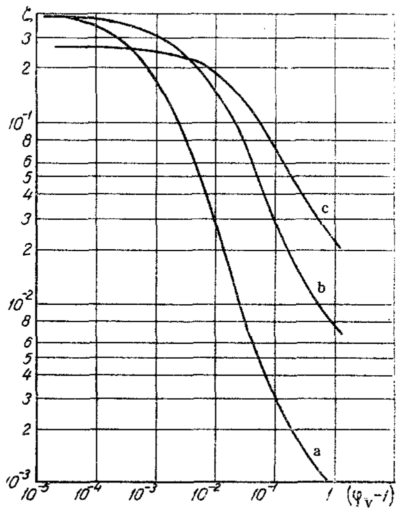

Fig. 2. Coefficient ζ as a function of $(\varphi_V - 1)$ and τ_g for $\bar{z} = 1$; x = 0.9; $\nu_0 = 0.6$; $R_1 = 1$; $R_2 = 0$: a) $\tau_g = 10^2$; b) 10; c) 1.

The greatest influence of ν_0 on the coefficient ζ is observed in the following cases: 1) in the region $0 \le \nu_0 \le 1$ at z = 0; 2) in the region $\nu_0 > 1$ at values of $\bar{c}_V$ corresponding to $\zeta = \zeta_{max} = 1$. At a considerable distance from the initial interval of motion of the particle ν_0 almost ceases to affect the coefficient ξ (Fig. 3, curves 1–5). In this case the calculation is based on equation (19).

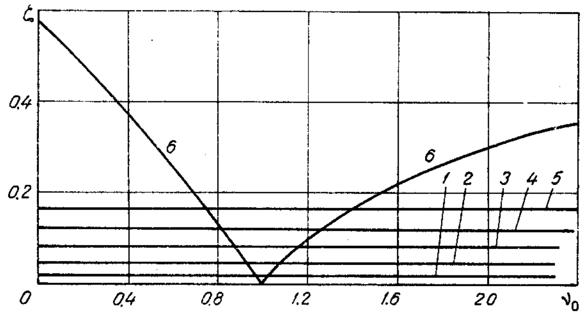

Fig. 3. Coefficient ξ as a function of ν_0 for $R_1 = 1$; $R_2 = 0$: 1–5) $\nu = \nu_{V1}$, $\bar{\tau}_g = 10$; 1) x = 0.9; 2) 0.8; 3) 0.7; 4) 0.6; 5) 0.5; 6) $(c_V - 1) = 10^{-4}$; $\bar{\tau}_g = 10^{-4}$; x = 0.5–0.9.

A typical graph of the coefficient ξ as a function of τ_g is presented in Fig. 4. The maximum of ξ occurs at $\bar{\tau}_g = 0.01–1$.

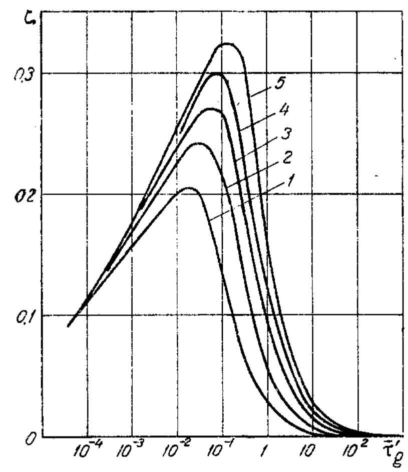

Fig. 4. Coefficient ξ as a function of $\bar{\tau}_g$ and x for $\nu_0 = 0.6$; $(c_V - 1) = 10^{-2}$; $R_1 = 1$; $R_2 = 0$: 1) x = 0.9; 2) 0.8; 3) 0.7; 4) 0.6; 5) 0.5.

At constant $(\varphi_V - 1)$ Fig. 4 gives the dependence of ξ on the Stokes number. The extremal character of the dependence of ξ on $\bar{\tau}_g$ when the other parameters are fixed is explained as follows.

We represent the work $\bar{A}_f$ as the product of the dimensionless friction power $\bar{N}_f$ and the dimensionless time $\bar{\tau}$ during which $\bar{N}_f$ "acts." In particular, for $\nu_0 = \nu_{Vi}$, $R_1 = 1$ and $R_2 = 0$, as follows from (18),

$$\bar{N}_f = \frac{(1-x)(1-\nu_{Vi})^2}{\nu_{\pi i}} \tag{28}$$

and

$$\bar{\tau} = \frac{(\bar{c}_V - 1)(\bar{c}_V + 1)}{\bar{\tau}_g}. \tag{29}$$

If x and $\bar{c}_V$ are fixed, then, as $\bar{\tau}_g$ varies from zero to infinity, $\bar{N}_f$ increases monotonically from zero to infinity, whereas $\bar{\tau}$ decreases monotonically from infinity to zero. At small $\bar{\tau}_g$ the time $\bar{\tau}$ is considerable in magnitude, but $\bar{N}_f$

is extremely small. In the region of large $\overline{\tau}_g$, the friction power is quite large, but the time during which it "acts" is negligibly small. Consequently, at both large and small $\overline{\tau}_g$ $\overline{A}_f \to 0$. Since on the interval $0 \le \overline{\tau}_g < \infty$ the work $\overline{A}_f$ is expressed as a product of an increasing function $\overline{N}_f$ and a decreasing function $\overline{\tau}$, the dependence of $\overline{A}_f$ on $\overline{\tau}_g$ has a maximum. Qualitatively, the variation of the coefficient ν in (14) resulting from the variation of $\overline{\tau}_g$ does not affect the dependence of ζ on $\overline{\tau}_g$. Consequently, $\zeta(\overline{\tau}_g)$, like $A(\overline{\tau}_g)$, has a maximum at some value of $\overline{\tau}_g$.

As the above analysis has shown, the effect of all the dimensionless similarity criteria on ζ is confined to certain ranges of variation. Outside these ranges the coefficient ζ is almost independent of the criteria in question. The result obtained is nothing other than the widely known effect of "degeneracy of similarity criteria" [1]. The above relations make it possible to determine the limits of the ranges of variation for a given degree of influence of the criteria on the quantity ζ.

It is worthwhile estimating the effect of particles present in the flow on the vapor velocity for a one-dimensional adiabatic two-phase flow. If the volume occupied by the liquid phase is much less than the vapor volume, and we disregard the effect of interphase heat and mass transfer, we can express the relative vapor velocity as follows:

$$\overline{c}_n = \sqrt{\frac{2}{(c_v)_0^2}\,[(i_v)_0 - i_v] + 1 - \overline{A}}, \tag{30}$$

where

$$\overline{A} = 2A/(c_v)_0^2 = (1-x)\,[(\overline{c}_v\,\nu)^2 - \nu_0^2] + \overline{A}_f.$$

Equation (30) is the formula known from the gasdynamics of one-phase media supplemented by a term that takes into account the forces of mechanical interaction between the vapor and the liquid. Since at the beginning of the calculation $\overline{c}_s$ and, hence, $\overline{A}$ are unknown, $\overline{c}_v$ is calculated from (30) by successive approximations.

The analysis of the energy losses associated with mechanical interaction between the vapor and the liquid shows that the loss coefficient ζ is a complex function of a series of dimensionless criteria: ν_0, $\overline{\tau}_g$, x, etc. In a number of cases ζ reaches a considerable value and then the class of losses in question should be taken into account in two-phase channel calculations. The formulas obtained above make it possible to find the qualitative and quantitative relations between ζ and the principal similarity criteria of the interphase energy transfer process.

NOTATION

A is the work; A_1 is the work done by the gas in absolute motion; A_f is the absolute value of the kinetic energy losses in gas-liquid system generated by friction at the phase interface; $\overline{A} = 2A/(c_v)_0^2$ is the dimensionless work; c is the velocity; $\overline{c} = c/(c_v)_0$ is the dimensionless velocity; $F = F_1 + F_2$ is the force of interaction between particle and gas; F_1 and F_2 are components of the force F associated with the dissipation of kinetic energy (F_1) and reversible interphase energy transfer in the gas-liquid system; E is the kinetic energy per unit of mass flow rate of the two-phase medium; i is the enthalpy; N is the power; $N_1 = Fc_v$; $N_2 = Fc_s$; $N_3 = F(c_v - c_s)$; $R_1 = 1 + (S/\rho)$, $R_2 = (1+S)/\overline{\rho}$; S is the total additional mass coefficient; $(1 - x)$ is the mass concentration of liquid phase; z is the linear coordinate; Δz is the distance traveled by the particle; $\overline{z} = z/\Delta z$ is the dimensionless coordinate ($0 \le \overline{z} \le 1$); $\nu = c_s/c_v$ is the slip factor; $\nu_{v1} = 2(1 + R_2\overline{\tau}_g)/(1 + \sqrt{\Delta})$; $\nu_{v2} = (1 + \sqrt{\Delta})/(-2R_1\overline{\tau}_g)$; $\Delta = 1 + 4R_1\overline{\tau}_g(1 + R_2\overline{\tau}_g)$; ρ is the density; $\overline{\rho} = \rho_s/\rho_v$; τ is the time; τ_g is the generalized Stokes number; $\overline{\tau}_g = \tau_g(d\overline{c}_v/d\overline{z})$; $\varphi = \ln\overline{c}_v$. Subscripts: s—liquid (solid) phase; 0—parameters at $\overline{z} = 0$; v—vapor (gas) phase; f—friction.

REFERENCES

1. A. A. Gukhman, Introduction to the Theory of Similarity [in Russian], Izd. Vysshaya shkola, Moscow 1963.

15 January 1968

Power Engineering Institute, Moscow